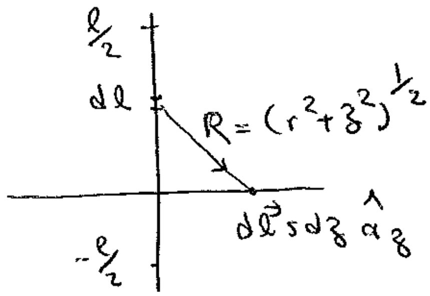
Problem 5.24

a)



$z\Big|_{-\ell/2}^{\ell/2}$

$$\vec{A} = \frac{\mu}{4\pi}\int \frac{I\,d\vec{\ell}}{R}$$

$$\vec{A} = \frac{\mu I}{4\pi}\int_{-\ell/2}^{\ell/2} \frac{dz}{(r^2+z^2)^{1/2}}\,\hat{a}_z$$

$$A_z = \frac{\mu I}{4\pi}\ln\left(z + \sqrt{r^2+z^2}\right)\Big|_{-\ell/2}^{\ell/2}$$

$$A_z = \frac{\mu I}{4\pi}\ln\left(\frac{\frac{\ell}{2} + \sqrt{\frac{\ell^2}{4}+r^2}}{-\frac{\ell}{2} + \sqrt{\frac{\ell^2}{4}+r^2}}\right) \Rightarrow \vec{A} = \frac{\mu I}{4\pi}\ln\left(\frac{\ell+\sqrt{\ell^2+4r^2}}{-\ell+\sqrt{\ell^2+4r^2}}\right)\hat{a}_z$$

b) $\vec{B} = \vec{\nabla}\times\vec{A}$

$$\vec{\nabla}\times\vec{A} = -\frac{\partial A_z}{\partial r}\hat{a}_\varphi = -\frac{dA}{dr}\hat{a}_\varphi$$

$$-\frac{dA_z}{dr} = \frac{-\mu I}{4\pi}\,\frac{1}{\left(\frac{\ell+\sqrt{\ell^2+4r^2}}{-\ell+\sqrt{\ell^2+4r^2}}\right)} \times \frac{d}{dr}\left(\frac{\ell+\sqrt{\ell^2+4r^2}}{-\ell+\sqrt{\ell^2+4r^2}}\right)$$

$$-\frac{dA_z}{dr} = \frac{\mu I}{4\pi}\,\frac{8r\ell}{(\ell^2+4r^2)^{1/2}}\,\frac{1}{4r^2} = \frac{\mu I \ell}{2\pi r\sqrt{\ell^2+4r^2}}$$

$$\Rightarrow \vec{B} = \frac{\mu I \ell}{2\pi r\sqrt{\ell^2+4r^2}}\hat{a}_z \text{ Tesla}$$

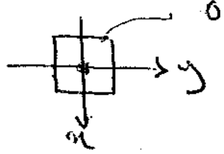
Problem 5.25

$$\vec{A} = 5\cos(\pi y)\,\hat{a}_x + (2+\sin\pi x)\,\hat{a}_z$$

a) $$\vec{B} = \vec{\nabla}\times\vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{a}_z$$

$$0 \qquad 0 \qquad\qquad 0 \qquad \pi\cos(\pi x) \qquad\qquad 0 \qquad -5\pi\sin\pi y$$

$$\rightarrow \vec{B} = -\pi\cos\pi x\,\hat{a}_y + 5\pi\sin\pi y\,\hat{a}_z$$

b) 

0.25m each side

$$d\vec{s} = dx\,dy\,\hat{a}_z$$

$$\Phi = \int_s \vec{B}\cdot d\vec{s} = \int_{-0.125}^{0.125}\int_{-0.125}^{0.125} 5\pi\sin\pi y\,dx\,dy$$

$$\Phi = 0.25(5\pi)\int_{-0.125}^{0.125}\sin\pi y\,dy$$

$$\Phi = \frac{5\pi}{4}\cos\pi y\Big|_{0.125}^{-0.125} = \frac{5\pi}{4}\left[\cos\left(-\frac{\pi}{8}\right) - \cos\left(\frac{\pi}{8}\right)\right]$$

$$\Phi = 0$$

c) $$\Phi = \oint_l \vec{A}\cdot d\vec{l} = -\int_{-\frac{1}{8}}^{1/8} A_x\Big|_{y=\frac{1}{8}}(-dx) + \int_{-\frac{1}{8}}^{\frac{1}{8}} A_y\Big|_{x=-\frac{1}{8}}(-dy) + \int_{-\frac{1}{8}}^{1/8} A_x\Big|_{y=-\frac{1}{8}}(+dx) + \int_{-\frac{1}{8}}^{\frac{1}{8}} A_y\Big|_{x=\frac{1}{8}}(+dy)$$

$$= -\int_{-\frac{1}{8}}^{1/8} 5\cos\left(\frac{\pi}{8}\right)dx + 0 + \int_{-\frac{1}{8}}^{\frac{1}{8}} 5\cos\left(\frac{-\pi}{8}\right)dx + 0$$

$$= -5\cos\frac{\pi}{8}\left[\frac{1}{8} - \left(-\frac{1}{8}\right)\right] + 5\cos\left(-\frac{\pi}{8}\right)\left[\frac{1}{8} - \left(-\frac{1}{8}\right)\right]$$

$$= \frac{-5}{4}\cos\frac{\pi}{8} + \frac{5}{4}\cos\left(\frac{-\pi}{8}\right)$$

$$\Rightarrow \quad \Phi = 0$$

Problem 5.26

$$\vec{J} = J_0 \hat{a}_z \frac{A}{m^2} \Rightarrow \vec{A} = \frac{-\mu_0 J_0}{4}(x^2+y^2)\hat{a}_z \quad \frac{Wb}{m}$$

$$\vec{A} = A_z \hat{a}_z$$

a) $\vec{\nabla}^2 \vec{A} = \nabla^2 A_z = \dfrac{\partial^2 A_z}{\partial x^2} + \dfrac{\partial^2 A_z}{\partial y^2} + \dfrac{\partial^2 A_z}{\partial z^2}$

$$= -\frac{\mu_0 J_0}{4}(2+2+0) = -\mu_0 J_0$$

b) $\vec{B} = \vec{\nabla} \times \vec{A} \qquad \vec{\nabla} \times A_z \hat{a}_z = -\dfrac{\mu_0 J_0}{2}(y\hat{a}_x - x\hat{a}_y)$

$$\vec{B} = \frac{-\mu_0 J_0}{2}(y\hat{a}_x - x\hat{a}_y)$$

$$\vec{H} = \frac{\vec{B}}{\mu} \rightarrow \vec{H} = -\frac{J_0}{2}(y\hat{a}_x - x\hat{a}_y)$$

c) $\oint_l \vec{H}\cdot d\vec{l} = I = \int_S \vec{J}\cdot d\vec{s}$

choosing a circular path in the x-y plane, $d\vec{l} = r\,d\varphi\,\hat{a}_\varphi$

$H = H_\varphi \hat{a}_\varphi$

$ds = r\,dr\,d\varphi\,\hat{a}_z$

$$\int_0^{2\pi} H_\varphi r\,d\varphi = H_\varphi \int_0^{2\pi} r\,d\varphi = 2\pi r H_\varphi$$

$$\int \vec{J}\cdot d\vec{s} = \int_0^{2\pi}\int_0^r J_0 r\,dr\,d\varphi = J_0 \frac{r^2}{2} 2\pi = \pi J_0 r^2$$

$$\Rightarrow H_\varphi = \frac{\pi J_0 r^2}{2\pi r} = \frac{J_0}{2} r \qquad \vec{H} = \frac{J_0}{2} r \hat{a}_\varphi$$

$$\vec{H} = \frac{J_0}{2}(x^2+y^2)^{1/2}\hat{a}_\varphi \quad \text{but } \hat{a}_\varphi = -\sin\varphi\,\hat{a}_x + \cos\varphi\,\hat{a}_y$$

$$= -\frac{y}{r}\hat{a}_x + \frac{x}{r}\hat{a}_y$$

$$\vec{H} = \frac{J_0}{2}(x^2+y^2)^{1/2}\left(\frac{-y}{(x^2+y^2)^{1/2}}\hat{a}_x + \frac{x}{(x^2+y^2)^{1/2}}\hat{a}_y\right)$$

$$\rightarrow \vec{H} = -\frac{J_0}{2}(y\hat{a}_x - x\hat{a}_y)$$

5.29

Fe $\quad 8.5\times10^{28}$ atoms / meter^3

$m_s = 9.27\times10^{-24}$ Ampers-meter^2

The electron spin magnetic moments yield 1.5 Tesla to B

dimension : $\left(\frac{\text{atoms}}{m^3}\right)\times\left(\frac{A\cdot m^2}{\text{electron}}\right)\times\left(\frac{\#\ \text{electrons}}{\text{atom}}\right) = \frac{A}{m}$

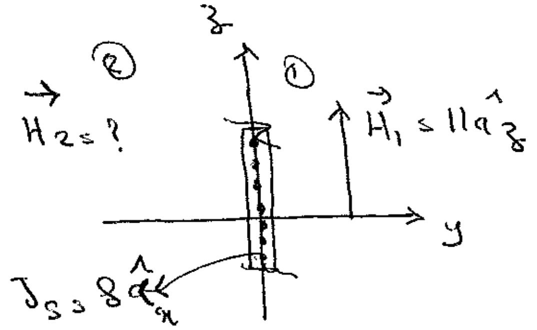
$$M = \left(8.5\times10^{28}\ \frac{\text{atoms}}{m^3}\right)\left(9.27\times10^{-24}\ A\cdot m^2\right) N\left(\frac{\text{electrons}}{\text{atom}}\right)$$

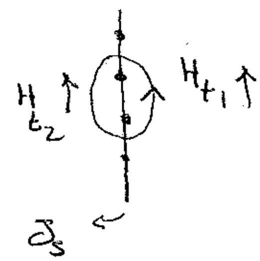
$$B = \mu_0 M \Rightarrow M = \frac{B}{\mu_0}$$

$$\rightarrow N = \frac{1.5}{4\pi\times10^{-7}}\left(\frac{1}{(8.5\times10^{28})(9.27\times10^{-24})}\right)\frac{\text{electrons}}{\text{atom}}$$

$$\Rightarrow N = 1.51\ \frac{\text{electrons}}{\text{atom}}$$

Problem 5.31





$$\oint \vec{H}\cdot\vec{dl} = J_s l$$

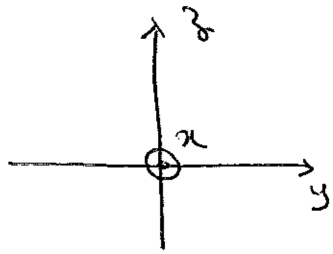
$$H_{t_1} l - H_{t_2} l = J_s l$$

$$\Rightarrow H_{t_1} - H_{t_2} = J_s$$

$$11 - H_{t_2} = 8$$

$$H_{t_2} = 3 \quad \Rightarrow \vec{H}_2 = 3\,\hat{a}_z$$

Problem 5.33



① air $\vec{B}_1 = 4\hat{a}_x - 6\hat{a}_y + 8\hat{a}_z$

$\mu = \mu_0$

$\rightarrow$ $z=0$ plane is the boundary

② iron $\mu = 5000\mu_0$

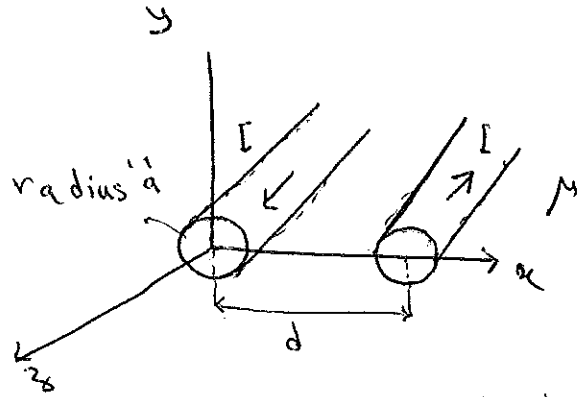
with no J_s on the boundary $\rightarrow H_{t1} = H_{t2} \rightarrow \frac{B_{t1}}{\mu_1} = \frac{B_{t2}}{\mu_2}$

Also, $B_{n1} = B_{n2}$ ($\hat{a}_z$ is normal to the boundary)

$\Rightarrow \vec{B}_2 = 4\left(\frac{5000}{1}\right)\hat{a}_x - 6\left(\frac{5000}{1}\right)\hat{a}_y + 8\hat{a}_z$

$B_2 = 20{,}000\,\hat{a}_x - 30{,}000\,\hat{a}_y + 8\hat{a}_z$

Problem 5.35



For the conductor along the z axis;

$$L = \frac{\Phi}{I} = \frac{\int_s \vec{B} \cdot d\vec{s}}{I} = \frac{\int_0^{\ell} \int_a^{d-a} \frac{\mu I}{2\pi r}\, dr\, dz}{I}$$

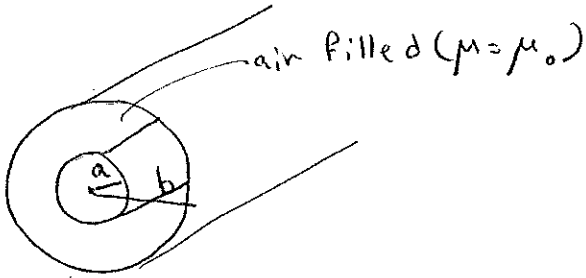
$$L_1' = \frac{L_1}{\ell} = \frac{\mu}{2\pi} \ln(d-a)$$

The other line along $x = d$, $y = 0$ yields an equal amount, so

$$L_2' = L_1' \qquad L' = L_1' + L_2' = 2L_1'$$

$$L' = \frac{\mu}{\pi} \ln\left(\frac{d-a}{a}\right) \text{ Henrys/meter}$$

Problem 5.37



$\ell = 3\,m$

$a = 5\,cm$

$b = 10\,cm$

$$W = \frac{1}{2} L I^2 \qquad L = \frac{\int_s \vec{B} \cdot \vec{ds}}{I} = \int_0^{\ell} \int_a^b \frac{\mu_o I}{2\pi r}\, dr\, dz$$

$$L = \frac{\mu_o L}{2\pi} \ln\left(\frac{b}{a}\right)$$

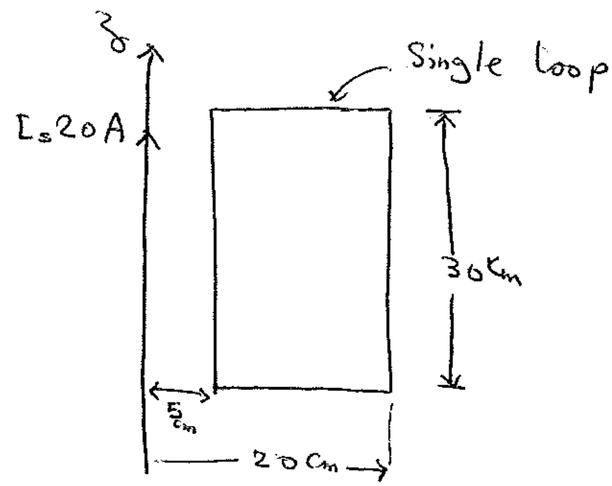
$$W = \frac{\mu_o \ell I^2}{4\pi} \ln\left(\frac{b}{a}\right)$$

$$W = \frac{3 I^2}{\cancel{4\pi}} \ln\left(\frac{10}{5}\right)(\cancel{4\pi} \times 10^{-7})$$

$W = 2.08 \times 10^{-7} I^2$ Joules

$W = 208\, I^2$ nJ

Problem 5.38



$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{a}_\varphi \qquad \vec{ds} = dr\,dz\,\hat{a}_\varphi$$

$$\Phi = \int_s \vec{B}\cdot\vec{ds} = \frac{\mu_0\, 20}{2\pi}\int_0^{0.3}\int_{0.05}^{0.2}\frac{1}{r}\,dr\,dz$$

$$\Phi = 40\times10^{-7}\cdot(0.3)\ln(4) \text{ webers}$$

$$= 16.64\times10^{-7}$$

$$\Phi = 1.664\,\mu \text{ webers}$$